Uniformity of the Magnetic Field Produced by a Cosine Magnet with a Superconducting Shield*

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Abstract

The well-known cosine theta magnet is analysed for magnetic field uniformity. We surround the magnet with a superconducting shield and study the impact on the uniformity. We find an equation for the uniformity as a function of principal magnet and shield parameters. We also find that penetrations in the shield and indeed the presence of the shield do not significantly alter the uniformity properties of the cosine theta magnet.

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1 Introduction

Experiments often demand a uniform magnetic field over an extended region of space for experiments. Several configurations of magnets have been built over the years to match this task. One that seems to enjoy particular favor for fields perpendicular to the axis of a cylinder is the so called $\cos(\theta)$ magnet. This is a cylindrical magnet whose wires are on the surface of a cylinder in a cosine distribution of the azimuthal angle θ around the cylinder. This type of magnet is well known amongst physicists. However, the literature on its properties is very slim. The reader may look in the literature for a calculation of the different multipole strengths [1], a practical design and construction [2] and [3], a comparison to different experimental [4] and theoretical designs [5]. However, the literature does not discuss the uniformity of the magnetic field, a gap we attempt to bridge by presenting both an analytical and numerical model of the magnet.

This paper is motivated by the experimental need for such a magnetic field in a Neutron Electric Dipole Moment (EDM) experiment at the Los Alamos National Laboratory [6]. The experiment plans to use a superconducting quantum interference device (SQUID) coupled to a large superconducting pick-up loop as part of its magnetometry. This loop must be shielded from stray fields and so a superconducting shield is placed around the cylinder carrying the magnet and pick-up loop. The three questions to be answered are: What are the dimensions of the magnet required to produce a uniformity of 0.1 percent over the dimensions ($10cm$ radius and $10cm$ height) of the measuring cell? How large is the deviation from this uniformity introduced by a superconducting shield? What is the effect of penetrations in the shield?

2 The Cosine Magnet

We begin by selecting the current density to be in the aforementioned cosine distribution together with a delta function which indicates the presence of

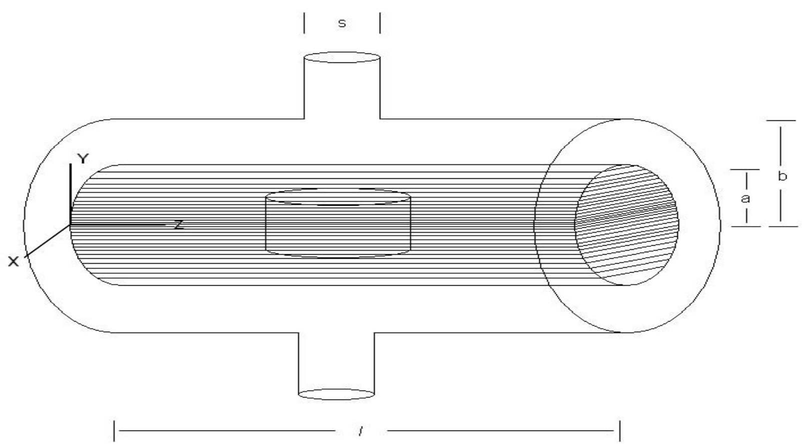

Fig. 1. The experimental schematic of the EDM experiment.

In Cartesian coordinates in which the **k** axis is along the axis of the cylinder, we have

$$\mathbf{J}(\mathbf{x}) = I \frac{x}{\sqrt{x^2+y^2}} \delta\left(x^2+y^2-a^2\right)\mathbf{k} \tag{1}$$

where I is the current density (Am^{-2}) and a is the radius of the magnet, see Fig. 1. This form of the current density ignores the endcaps and is only valid when the length of the magnet is significantly larger than a. Then we may obtain the vector potential via (for any basic result of classical electromagnetism, please see [7])

$$\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d^3x' \tag{2}$$

The integral over y' may be done using the delta function to get

$$\mathbf{A}(\mathbf{x}) = \frac{\mu_0 I \mathbf{k}}{4\pi a} \int \frac{x' dx' dz'}{\left[x^2 + y^2 + z^2 + a^2 + z'^2 - 2xx' - 2zz' - 2y\sqrt{a^2 - x'^2}\right]^{\frac{1}{2}}} \tag{3}$$

To find the magnetic field, we must take the curl of the vector potential. We may also do the integral over dx' from 0 to l, the length of the magnet to get

$$\begin{aligned} \mathbf{B}(\mathbf{x}) = &-\frac{\mu_0 I \mathbf{i}}{2\pi a} \int_{-a}^{a} \frac{dx' x' \left(y - \sqrt{a^2 - x'^2}\right)}{F - z^2} \left[\frac{l - z}{\sqrt{l^2 - 2lz + F}} + \frac{z}{\sqrt{F}}\right] \\ &+ \frac{\mu_0 I \mathbf{j}}{2\pi a} \int_{-a}^{a} \frac{dx' x' (x - x')}{F - z^2} \left[\frac{l - z}{\sqrt{l^2 - 2lz + F}} + \frac{z}{\sqrt{F}}\right] \end{aligned} \tag{4}$$

where

$$F = x^2 + y^2 + z^2 + a^2 - 2xx' - 2y\sqrt{a^2 - x'^2} \tag{5}$$

We note that the magnetic field has no component along the axis of the cylinder. Also, we note that since the integrand of the magnetic field in the $\mathbf{i}$ direction is nearly odd; this component of the magnetic field is, in general, small. Indeed numerical calculations verify that it is almost neglegible. Thus the field will be almost perfectly in the $\mathbf{j}$ direction. The remaining integral can not be done in general, however if we consider the field on axis of the cylinder, we get

$$\mathbf{B}(0,0,z) = \frac{-\mu_0 I}{6\pi}\left[\frac{l-z}{\sqrt{(l-z)^2+a^2}} + \frac{z}{\sqrt{z^2+a^2}}\right]\mathbf{j} \tag{6}$$

which is almost perfectly constant over the middle region of the cylinder as long as the length is significantly larger than the radius squared. When the field is plotted as a function of distance along the horizontal and vertical axes, we get a region of uniformity in the center as we would expect. See Fig. 2, which is plotted using the numerical simulation about to be discussed but which agrees completely with the analytical results above.

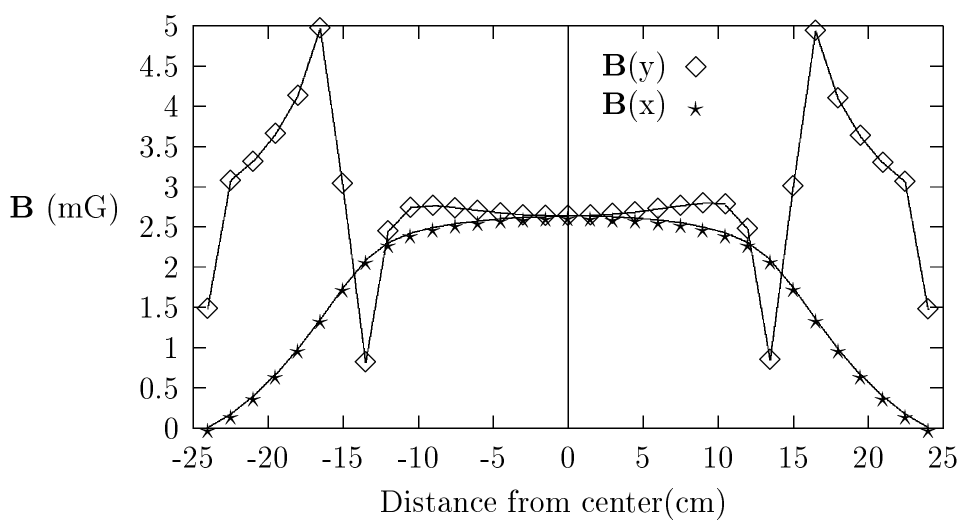

Fig. 2. Magnetic field strength as a function of distance in the horizontal and vertical directions of the cosine magnet. The current has been adjusted to give a

central value of $\mathbf{B}$ near $2.5mG$. This plot is for a magnet of length $1.5m$ and of radius $0.25m$ with a shield radius of $0.3m$, see Fig. 1. The region of uniformity in the center is where an experiment could be performed.

3 The Superconducting Shield

The boundary condition at the surface of a superconducting shield is the vanishing of the vector potential. In effect, we induce a current on the shield whose vector potential exactly cancels that of the magnet at the surface of the shield. This induced vector potential changes the magnetic field inside the magnet. The induced current is

$$\mathbf{J}(\mathbf{x}) = \frac{4\pi}{\mu_0}\mathbf{A}(\mathbf{x})\delta\left(x^2 + y^2 - b^2\right) \tag{7}$$

where b is the radius of the shield, see Fig. 1. Again, this form of the current density is only valid if the length of the shield is significantly larger than its radius. We find the vector potential via the same means to get

$$\mathbf{A}_s(\mathbf{x}) = \frac{\mu_0 I\mathbf{k}}{4\pi a}\int \frac{H(x', z')dx'dz'}{\left[x^2 + y^2 + z^2 + b^2 + z'^2 - 2xx' - 2zz' - 2y\sqrt{a^2 - x'^2}\right]^{\frac{1}{2}}} \tag{8}$$

where

$$H(x', z') = \int_{-a}^{a}\left[\frac{l - z' + \sqrt{l^2 - 2z'l + G}}{\sqrt{G} - z'}\right]x''dx'' \tag{9}$$

where

$$G = z'^2 + a^2 + b^2 - 2x'x'' - 2\sqrt{b^2 - x'^2}\sqrt{a^2 - x''^2} \tag{10}$$

The magnetic field is the curl of the vector potential, thus

$$\mathbf{B}_s(\mathbf{x}) = \frac{\mu_0 I}{4\pi a} \int \frac{\mathbf{J}(x', z') \left[(x - x')\,\mathbf{j} - \left(y - \sqrt{a^2 - x'^2}\right)\mathbf{i}\right] dx'dz'}{\left[x^2 + y^2 + z^2 + b^2 + z'^2 - 2xx' - 2zz' - 2y\sqrt{a^2 - x'^2}\right]^{\frac{1}{2}}} \tag{11}$$

and the total field is just the superposition of the two fields,

$$\mathbf{B}_{total}(\mathbf{x}) = \mathbf{B}_s(\mathbf{x}) + \mathbf{B}(\mathbf{x}) \tag{12}$$

When the relevant integrals are numerically computed, we find that the presence of the shield does not significantly change the uniformity properties of the cosine magnet to the level of one part in 10^4. This is true as long as $b - a > 3cm$. The dimensions of the magnet can be adjusted to achieve a uniformity of one part in 10^3 over the required region of space with the shield present.

4 Numerical Simulation

A real EDM experiment must incorporate individual wires and penetrations in the cylinders. Such complications requires numerically solving the Poison differential equation for the vector potential

$$\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J} \tag{13}$$

and then taking the curl to get the magnetic field, the endcaps are now included. The superconducting shield is incorporated by the boundary condition that the vector potential at the shield must vanish. This procedure is readily implemented using numerical techniques for solving partial differential equations. Only ten wires are required to make the difference between the above calculation and the numerical model totally insignificant.

We define the uniformity U to be

$$\begin{aligned} U^2 = & \frac{[\mathbf{B}(r_c,0,l/2) - \mathbf{B}(0,0,l/2)]^2}{3\mathbf{B}^2(0,0,l/2)} + \frac{[\mathbf{B}(0,L_c/2,l/2) - \mathbf{B}(0,0,l/2)]^2}{3\mathbf{B}^2(0,0,l/2)} \\ & + \frac{[\mathbf{B}(0,0,l/2+r_c) - \mathbf{B}(0,0,l/2)]^2}{3\mathbf{B}^2(0,0,l/2)} \end{aligned} \tag{14}$$

in terms of the magnetic field $\mathbf{B}(x, y, z)$, where L_c and r_c are the height and radius of the cell. For different sizes of the magnet, shield and region of uniformity (where an experimental cell would be placed), the uniformity was calculated with the numerical model. Using this data, is was possible to derive an empirical equation for the uniformity U given the radius of the magnet r_m, radius of the shield r_s and length of the magnet and shield L_m

$$U = \exp\left(13.45L_c + 1.37r_c - 13.1L_m - 34.4r_m - 24.59\right) \pm 66.7\% \tag{15}$$

This equation is valid as long as the independant variables are of reasonable magnitude (greater than $10cm$ and less than $3m$) and the configuration is possible, see Fig. 1. The uncertainty arises from many factors such as inaccuracies in the solution of Poison's equation and in curve-fitting techniques. This equation is a useful guide to anyone interested in manufacturing a cosine magnet with a superconducting shield. It gives a simple estimate of the field uniformity for a particular configuration in a straightforward manner.

The needs for the EDM experiment are a field of $1mG$ with a deviation of the order of $1\mu G$ over a cylindrical region $20cm$ in length with a $10cm$ diameter. If the input current density is $I = 1Am^{-2}$, both cylinders are $1.5m$ in length and the radii are 0.3 and $0.6m$ respectively, this field profile is achieved even in the presence of the superconducting shield.

Further simulations were carried out which included one or two holes in the shield, with cylindrical superconducting pipes leading to them, of a diameter and length of $12cm$ for leads necessary for the experiment, see Fig. 1. In both cases, the program showns that the deviations of the field to the case of no holes is of the order of tenths of microgauss. Thus, holes in the shield, with superconducting pipes do not influence the field much.

It is of considerable interest for the EDM experiment to know what effect the penetrations in the shield have upon external field leakage. If a substantial leakage is detected, the SQUID's may not be able to measure the tiny fields which constitute the experimental signal. If B_p is the magnitude of the field at the end of the superconducting pipe of one of the two symmetric penetrations and $B(d)$ is the magnitude at a distance d from the end of this pipe towards the center of the magnet, then we define the attenuation length of the field magnitude d_0 via

$$B(d) = B_p e^{-d/d_0} \tag{16}$$

Simulations were conducted with different diameters of the two penetrations s (the length of the superconducting pipes leading to the penetrations were of fixed length $9.6cm$) and an empirical relationship between d_0 (cm) and s (cm) was discovered

$$d_0 = 4.24 \ln(0.278s + 1) \tag{17}$$

This shows that the distance for the field to drop in magnitude by a factor e, the attenuation length, increases with the diameter of the penetrations. As expected, when the penetrations are not there ($s = 0$), the attenuation length vanishes and we have no leakage fields. However, an order of magnitude reduction in B is achieved (for $s = 5$ cm) in 39.5 cm, a significant distance. Therefore, the external field shielding *outside* the superconducting shield must eradicate most of the fields.

5 Conclusion

The cosine magnet can produce a magnetic field for the Los Alamos EDM experiment with the required uniformity over a desired region of space. The primary idea is to make the magnet sufficiently large. A formula which quantifies "sufficiently large" was presented as equation (15) and demonstrates the dimensions are still practical. Penetrations and the presence of a superconducting shield do not influence the uniformity properties of the cosine magnet to an appreciable degree. However, penetrations greatly weaken the shielding efficiency of the superconducting shield.

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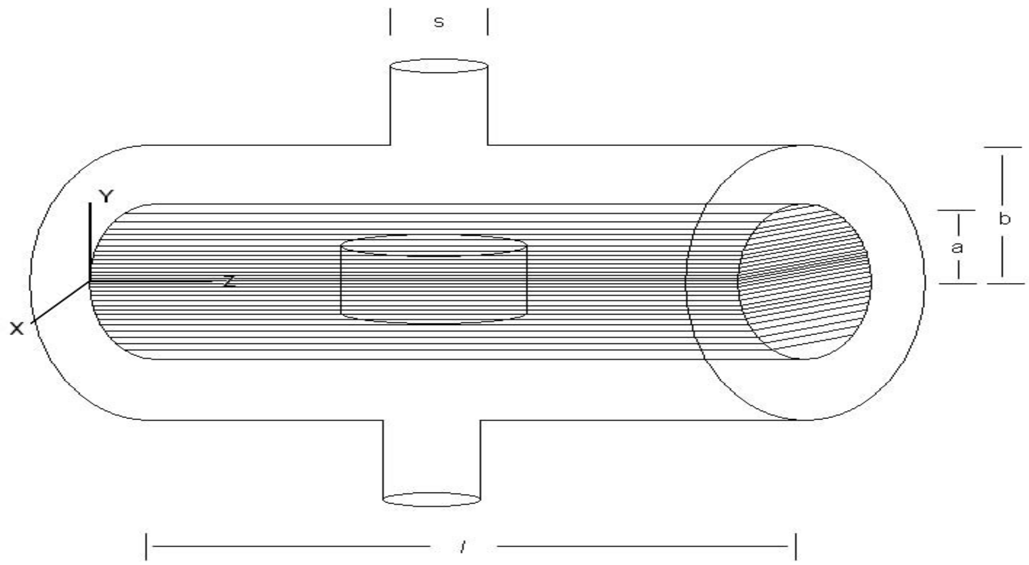
Figure captions:

Fig. 1: The experimental schematic of the EDM experiment.

Fig. 2: Magnetic field strength as a function of distance in the horizontal and vertical directions of the cosine magnet. The current has been adjusted to give a central value of $\mathbf{B}$ near $2.5mG$. This plot is for a magnet of length $1.5m$ and of radius $0.25m$ with a shield radius of $0.3m$, see Fig. 1. The region of uniformity in the center is where an experiment could be performed.



Better quality picture of Fig. 1.